Modern Numerical Hydrodynamics and the Evolution of Dense Interstellar Medium

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RESUMEN

Se discuten los problemas relacionados con las simulaciones hidrodinámicas en medios de alta densidad y se describe un nuevo código numérico, basado en la técnica de refinamiento de mallas, para simular la evolución de remanentes de supernova en estos ambientes.

ABSTRACT

We discuss some typical problems related to numerical hydrodynamics of a dense interstellar medium. A newly developed hydrodynamical code based on adaptive mesh refinement technique is presented and applied to simulate the evolution of a supernova remnant in a high-density medium. Advantages of this new approach in comparison to commonly used hydrodynamical methods are briefly discussed.

1. INTRODUCTION

There is a growing evidence that some massive stars end their lives as supernovae while being surrounded by a dense circumstellar medium (Fransson 1994, Terlevich 1994). Most of the evidence comes from radio observations indicating that in some cases (SN 1979C, SN 1980K, SN 1986J) the progenitors were probably losing their envelopes, in dense stellar winds, shortly before the explosion (Van Dyk et al. 1996). In other cases (SN 1978K, SN 1988Z), a strong radio remnant becomes a powerful X-ray source only a few years after explosion (Ryder et al. 1993, Fabian & Terlevich 1996). In both cases, the most probable origin of the emission is the interaction of the supernova ejecta with an external medium.

To explain properties of the radio emission from supernova remnants (SNRs), Chevalier (1982) proposed the *mini-shell* model in which this emission originates from the Rayleigh-Taylor unstable shell formed in the region separating the reverse and forward supernova shocks. This model was successfully applied to the analysis of radio observations of several SNRs (see Weiler et al. 1996 for a summary). Hydrodynamical models of the ejecta-wind interaction phase (Chevalier & Blondin 1995) clearly show the rapid formation of a dense shell in the shocked ejecta behind the reverse shock. In this case the reverse shock becomes a source of the X-ray emission which is partially reprocessed by unshocked ejecta and partially absorbed by the shell (Fransson et al. 1996). However, in its present form the ejecta-wind interaction scenario fails to explain the very high ($\gtrsim 10^{41}$ erg s^{-1}) X-ray luminosity of SN 1988Z observed some 7 years after the explosion (Fabian & Terlevich 1996). There is also a compelling spectroscopic evidence for the existence of a dense ($n_{\mathrm{amb}} \gtrsim 10^6$ cm^{-3} circumstellar medium (Stathakis & Sadler 1991). Also, the slow decline rates of the broad-band emission (Turatto et al. 1993) gives indirect evidence for an additional source of the energy powering the remnant. As it has been shown analytically (Shull 1980, Wheeler et al. 1980) supernova remnants evolving in very dense ($n_{\mathrm{amb}} > 10^5$ cm^{-3}) medium become powerful X-ray emitters shortly after the blast wave begins to interact with the medium. This result has been confirmed later by hydrodynamical simulations (Terlevich et al. 1992).

2. NUMERICAL TECHNIQUE

A major problem in numerical studies of supernova remnants is the large range of spatial and temporal scales. For this reason it is difficult to use conventional methods of astrophysical hydrodynamics to study such

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a violent phenomena in a dense medium. Here we present simulations of a SNR evolving in a uniform medium, performed with an adaptive mesh refinement (AMR) technique.

The AMR can be regarded as an automatic procedure for the most efficient discretization of the equations describing the evolution of the physical system under study. Here we deal with Euler equations describing temporal and spatial evolution of the ejecta and the interstellar medium. Since Euler equations are hyperbolic, they are subject to the CFL condition setting the upper limit for the time step, and the maximum allowed time step changes proportionally to the size of the smallest zone of the grid. This is a basic problem for the algorithms that use moving meshes to increase the spatial resolution. Moreover, due to the CFL limit it is practically impossible to achieve proper resolution in two- and three- dimensional simulations. AMR builds a hierarchical (or nested) system of mesh patches located at different levels. Each level forms a single grid which in turn may contain one or more mesh patches. The patches may partially overlap or may occupy separate regions of the computational volume, but every mesh patch must be properly nested (covered) inside a patch located on the parent grid. The grids have different effective resolutions; the ratio of the zone size on the parent level to the zone size located on the child level (refinement ratio) is an integer constant typically equal to 2 or 4. Since Euler equations are conservation laws (of mass, momentum, and energy), the AMR algorithm must conserve physical quantities advected between different patches located on the same grid as well as between parent and child patches located on different grids.

Finally, the success of the AMR critically depends on the way in which the new meshes are created. Different strategies have been proposed to detect the regions which need to be refined in order to describe the evolution of the system accurately. For example, one may try, using a concept of Richardson extrapolation to estimate a local truncation error of the scheme (Berger & Colella 1988) but this method works well only in regions free of shocks, since modern numerical schemes always produce narrow shock profiles. On the other hand, one may use some empirical criteria if there is enough a priori information about the physical behavior of the system. For example, to refine the shocks one may try to detect the regions of compressive flows in which the local pressure contrast exceeds some threshold value. As the structure of the grids is dynamic, it consecutively adapts to changing flow conditions, and the high resolution is used only in those regions where the refinement criteria are satisfied. A full description of the AMR algorithm is certainly beyond the scope of this presentation. For present application we used the AMRA code (Plewa & Müller 1996) which closely follows a description of the AMR algorithm as given by Berger & Colella (1988); the hydrodynamical equations are solved using the PPM algorithm by Colella & Woodward (1984).

3. RESULTS AND DISCUSSION

We simulate the expansion of the supernova ejecta into a homogeneous, static medium in 1D assuming spherical symmetry. The mass of the ejecta is equal to $M_{\mathrm{ej}} = 5M_{\odot}$, and the total energy of the explosion is equal to $E_{\mathrm{ej}} = 1 \times 10^{51}$ erg. For the structure of the ejecta we adopt the profile given by Franco et al. (1993). We assume that the medium has solar metallicity. Radiative losses are calculated implicitly with a cooling function obtained with CLOUDY 84.09 (Ferland 1993). The minimum temperature of the gas is equal to $T_{\mathrm{min}} = 10^4$ K.

At the symmetry axis ($R = 0$), we impose a reflecting boundary condition while at the outer edge of the grid ($r = 1.79 \times 10^{17}$ cm) we allow for a free outflow. The base grid (level=1) contains 512 zones and we allow for two levels of refinement, with a refinement ratio between the grids equal to 4. Thus, the maximum resolution (on the third level) is equal to 8192 equidistantly distributed zones. The meshes are created in the regions where the density or pressure contrast exceeds 1.0 and 10.0, respectively. In other regions we control the quality of the solution by using a truncation error estimate, with a maximum allowed error of 0.01. In addition, we do not allow for a refinement in the region occupied by the unshocked ejecta. We use a Courant number equal to 0.8.

The results of the simulation are presented in Fig. 1.[4] Due to very high temperatures, the early expansion of the remnant proceeds almost adiabatically, but after two years (Fig. 1a) the first signs of cooling are visible in the post-main-shock region. At this time the finest grid covers the reverse shock ($R \approx 0.0145$ pc), the contact discontinuity ($R \approx 0.0175$ pc), and the forward shock ($R \approx 0.0195$ pc). After next 2.7 years (Fig. 1b), catastrophic cooling leads to the formation of a dense shell right behind the main shock, and the whole region between the contact discontinuity and the forward shock is covered by the finest grid. Still later ($t \approx 25$ yr) the shocked ejecta starts to cool rapidly and the inner shell forms (Fig. 1c). Note that also in this case the region of strong cooling is covered by the finest grid. Finally, both shells collapse to form a single structure (Fig. 1d).

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[4]The movies presented during the talk can be accessed at `http://www.MPA-Garching.MPG.DE/Hydro/AMR/movies/starbur.html`.
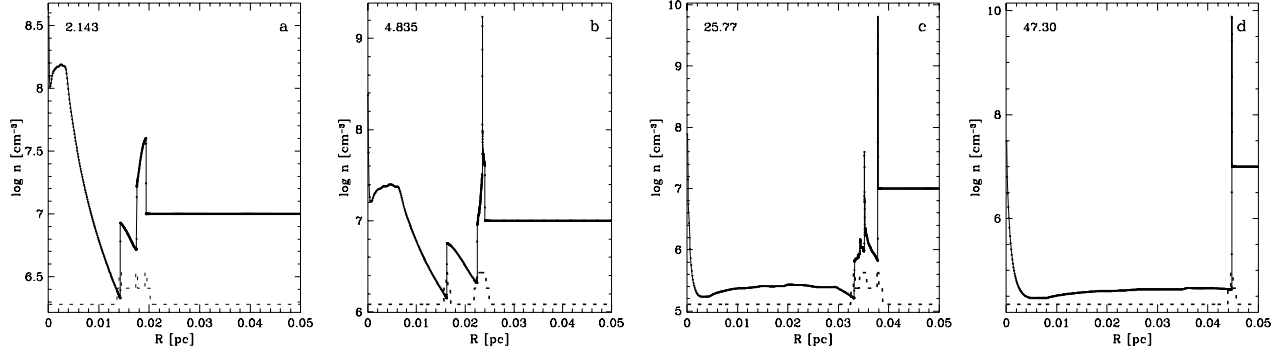
Fig. 1.— Temporal evolution of gas density and grid location (solid and dashed lines, respectively) for a SNR evolving in a dense medium. Evolutionary times (shown in upper-left corner of each panel) are given in years.

We note that although the present resolution is much too low to resolve some details (in particular both shells are unresolved), the AMR algorithm is able to adequately follow the evolution of all important flow structures. By calculating the mean volume occupied by the finest grid during the whole evolution, we find that in this particular case AMR offers an enormous saving of the CPU time (a factor of 100). This makes us to believe that, at least for problems where the number of structures to be resolved occupy a relatively small ($\leq 10\%$) fraction of the domain, the adaptive mesh refinement technique is an ideal tool. In the near future we plan to extend to two dimensions using a more realistic structure of the interstellar medium.

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